RETRIEVING FULL STIFFNESS MATRIX FROM EXPERIMENTAL CONDENSED MODELS

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ABSTRACT
This study aims to develop a system identification methodology for determining structural parameters of a linear dynamic system, taking into consideration practical constraints such as insufficient sensors. If the values at the damaged state are compared with the identified values at the undamaged state, damage detection and quantification can be carried out. The concepts of model condensation and mode shape expansion are invoked to avoid the need for complete measurement, and the recovery process to obtain the full set of parameters is formulated. All stiffness parameters in the entire system can be retrieved by extracting information using one of the two different approaches depending on whether the sensor can be repositioned. To estimate individual stiffness coefficient from the condensed stiffness matrices, the genetic algorithms approach is presented to accomplish the required optimization problem. Laboratory experiments involving an eight-storey frame model are carried out to illustrate the performance of the proposed method. The identification results presented in terms of the stiffness integrity index show that the proposed methodology is able to locate and quantify damage fairly accurately.

1. INTRODUCTION
With advancement in sensor and computer technology as well as the aging of many critical structures, renewed efforts can be seen in the area of structural health monitoring, focusing on the development of robust, efficient and accurate techniques especially for practical structures. Recent major earthquake and terrorist events also result in the need for tools in relation to damage assessment and safety evaluation of structures. One main approach in health
monitoring and damage detection methodology is through the use of system identification algorithms. This involves the assumption of a representative mathematical model of the real structure which characterizes the mechanical properties of the actual system, such as mass, stiffness and damping, so that the response can be computed for a given excitation. The inverse computation process is the use of known excitation and response data to estimate the structural parameters. Using the parameters at the healthy or some prior state of the structure under study as the base, the condition of the structure at any point in time can then be deduced through the inverse analysis.

The methods used for structural system identification can be classified under the time domain and the frequency domain. The simplest solution in the time domain approach is by the least-squares method [1, 2]. To reduce the effects of measurement noise, several methods have been proposed, e.g. Instrumental Variable [3], Maximum Likelihood [4] and Extended Kalman Filter or EKF [5, 6]. The most widely used model to represent the dynamics of a system in control theory is the first order matrix differential equation. Modal parameters typically include natural frequencies, modal damping ratios, and mode shapes are identified from a first-order state space model. Often in structural health monitoring and damage detection, it is necessary to obtain a second-order model with mass, stiffness and damping matrices. However, when retrieving physical parameters of the second-order model from the results of the first-order model [7-10], issues such as non-uniqueness of the solution have to be considered.

While considerable progress has been made owing to recent advances in instrumentation and computational capabilities, there are still many practical challenges in vibration-based damage detection. The number of sensors available for measurement is often limited and thus makes the task of identifying system with many unknown parameters difficult, particularly when attempting to identify the full system in one go. To this end, a reduced or condensed system may be identified corresponding to the number of sensors used. However, this does not necessarily give information on all the unknown parameters and the full model is unidentifiable. To resolve this problem and address the issue of incomplete measurement, a condensed model identification and recovery (CMIR) method has been developed [11, 12]. In the context of multi-storey buildings, this method aims to retrieve stiffness parameters of individual levels from the identified condensed stiffness matrices in a numerically efficient way.

Model condensation and mode shape expansion are considered in this study to eliminate the requirement of complete measurement. The well-known static condensation method of Guyan [13] eliminates unwanted or dependent coordinates to reduce the stiffness matrix size. Nevertheless, since dynamic effects are ignored in this method, the error can be large for dynamic problems with insufficient measurements representing the mass inertia. Many condensation methods have been proposed, for example, accounting for inertia forces partially by Kidder [14] and Miller [15]. Another method of reduction that may be considered as an extension of the static condensation method has been proposed, namely dynamic condensation [16]. O’Callahan et al. [17] proposed a new model reduction technique, which requires the full system eigenvectors corresponding to the set of modes of interest, and this is called System Equivalent Reduction Expansion Process (SEREP). Later, an approach using the eigenvectors from the reduced model was proposed by Papadopoulos and Garcia [18] to avoid using the full system eigenvectors. Recently, a new method for expanding incomplete experimental mode shapes has been presented by Chen [19] which considers the modelling errors in the analytical model and the uncertainties in the vibration modal data measurements.
The main objective of this paper is to develop a numerical strategy for retrieving the full stiffness matrix from experimental condensed models and structural damage assessment. A formulation and ideas to improve the identification performance for structural systems based on incomplete dynamic measurement are proposed. The proposed method is tested through experimental studies.

2. FORMULATION

The dynamic response of a $N$-DOF linear structural system can be represented by

$$
\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{L}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t)
$$

(1)

where $\mathbf{M}$, $\mathbf{L}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices of the structure, respectively, $\mathbf{q}$ is a displacement vector, and the overdot denotes differentiation with respect to time $t$. The equation of motion and measurement equations can be written in the following first-order discrete-time state space form

$$
\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)
$$

$$
\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k)
$$

(2)

where $\mathbf{x}(k)$ is a state vector, $\mathbf{y}(k)$ an output vector, and $\mathbf{u}(k)$ an input vector. Matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ represent time invariant system matrices. Under zero initial conditions, Eq. (2) can also be written in matrix form for a sequence of ‘$l$’ consecutive time steps as

$$
\mathbf{y}_{m \times l} = \mathbf{Y}_{m \times [(r+m)p + r]} \mathbf{V}_{[(r+m)p + r] \times l}
$$

(3)

where

$$
\mathbf{y} = \begin{bmatrix} y(0) & y(1) & y(2) & \ldots & y(p) & \ldots & y(l-1) \end{bmatrix}
$$

$$
\mathbf{Y} = \begin{bmatrix} \mathbf{D} & \mathbf{C}\mathbf{B} & \mathbf{C}\mathbf{A}\mathbf{B} & \ldots & \mathbf{C}\mathbf{A}^{p-1}\mathbf{B} \end{bmatrix}
$$

$$
\mathbf{V} = \begin{bmatrix} \mathbf{u}(0) & \mathbf{u}(1) & \mathbf{u}(2) & \ldots & \mathbf{u}(p) & \ldots & \mathbf{u}(l-1) \\
0 & \mathbf{v}(0) & \mathbf{v}(1) & \ldots & \mathbf{v}(p-1) & \ldots & \mathbf{v}(l-2) \\
0 & 0 & \mathbf{v}(0) & \ldots & \mathbf{v}(p-2) & \ldots & \mathbf{v}(l-3) \\
\vdots & \vdots & 0 & \ddots & \vdots & \ldots & \vdots \\
0 & 0 & \ldots & 0 & \mathbf{v}(0) & \ldots & \mathbf{v}(l-p-1) \end{bmatrix}
$$

in which $p$ is an integer such that $\mathbf{C}\mathbf{A}^k\mathbf{B} \approx 0$ for $k \geq p$. Having identified the observer Markov parameters $\mathbf{Y}$, the system’s Markov parameters $\mathbf{Y} = [\mathbf{D} \mathbf{C} \mathbf{B} \mathbf{C} \mathbf{A} \mathbf{B} \ldots \mathbf{C} \mathbf{A}^{p-1} \mathbf{B}]$ can
be retrieved through a simple recursive formula, namely Observer/Kalman filter Identification (OKID) [20].

In the Eigensystem Realization Algorithm (ERA), the generalized Hankel matrix is first formed, comprising the Markov parameters. The ERA process starts with the factorization of the first Hankel matrix using singular value decomposition, \( H(0) = R \Sigma S^T \) [21]. The triplet

\[
A = \Sigma^{-1/2} R_n^T H(l) S_n \Sigma^{-1/2} \quad B = \Sigma^{-1/2} S_n^T E_r \quad C = E_m^T R_m \Sigma^{-1/2}
\]

is a minimum realization where \( E_r = [I_{r,r} \ 0_{r,r} \ 0_{r,r} \ 0_{r,r} \ 0_{r,r}]^T \) with \( I \) denoting an identity matrix and \( 0 \) denoting a matrix whose elements are all zeros, and \( E_m \) is defined similarly.

The first-order model in Eq. (2) is written based on the second-order model in Eq. (1) and hence their eigenvectors are related through similarity transformed. The eigenvalues and eigenvectors of the first-order system can first be computed using \( A \) and then transformed and properly scaled to become the eigenvectors \( P \) and eigenvalues \( \Gamma \) of the structural model in Eq. (1). The mass, stiffness, and damping matrices can be obtained [10]

\[
M = (P \Gamma P^T)^{-1} \quad K = -(P \Gamma^{-1} P^T)^{-1} \quad L = -M P \Gamma^2 P^T M
\]

It is possible to detect damage by identifying storey stiffness values and comparing them with the corresponding values of the original (presumably undamaged) state. For this purpose, a simple stiffness integrity index is defined as

\[
S_i = \frac{K_d(i)}{K_u(i)}
\]

where \( K_d(i) \) and \( K_u(i) \) are the storey stiffness value of the \( i^{th} \) storey for the damaged state and undamaged state, respectively. The stiffness integrity index is 1 for no loss in stiffness and 0 for complete loss of stiffness.

### 2.1 Condensed model identification and recovery (CMIR) method

Full measurement is often impractical due to limited sensors and other practical constraints in instrumentation. As such, a condensed model based on incomplete measurement yields only the condensed stiffness matrices. The challenge is to determine individual stiffness parameters of the full system indirectly from the identified condensed stiffness matrix. In the CMIR method, all stiffness parameters in the entire system can be recovered by extracting sufficient information using two different approaches, namely the “repositioned sensor” approach and “fixed sensor” approach. In some cases, it is possible to shift the sensors to maximize the information available for structural identification and this approach is called the repositioned sensor approach. If it is not possible to shift the sensors, a novel way is to apply the fixed sensor approach and to deliberately ignore some sensors so as to obtain more information for the purpose of matrix recovery. The identification procedure for fixed sensor approach is as follows

(a) Identify the condensed stiffness matrix \( (n \times n) \) and mass matrix \( (n \times n) \) by OKID/ERA with conversion from first-order model to second-order model based on data from \( n \) sensors in the \( N \)-DOF system. Eigenvalues and eigenvectors are computed based on
the identified matrices. Model condensation is used to obtain the analytical condensed stiffness and mass matrix, and the corresponding eigenvalues and eigenvectors are computed.

(b) Repeat Step (a) as many times as possible by ignoring one sensor at a time. If necessary, repeat by ignoring two sensors or more until sufficient information is obtained. The number of ways to ignore \( m \) sensors is \(^m C_n\).

(c) Minimize the error functions as shown in Eq. (14). Note that it is possible to obtain more information than required (i.e. \( N \)) leading to an over-determined system.

\[ K_f \]

\[ K_d \]

\[ K_f \]

Figure 1. A 7-DOF shear building

To explain how more information can be obtained by ignoring some sensors in Step (b) described above, an illustration example is explained based on the static condensation method. Suppose 3 sensors are used to identify the 7-DOF structural system (Figure 1): one sensor each fixed at the 1\(^{st}\), 4\(^{th}\) and 7\(^{th}\) DOFs. If all 3 sensors are used, it is possible to identify a 3 x 3 condensed stiffness matrix, \( \bar{K}_I \) as follows.

\[
\bar{K}_I = \begin{bmatrix}
K_{11} & K_{12} & 0 \\
K_{21} & K_{22} & K_{23} \\
0 & K_{32} & K_{33}
\end{bmatrix}
\] (7)

where

\[
K_{11} = K_1 + K_2 - K_2^2 (K_3 + K_4) / K_{D1} \quad K_{12} = K_{21} = -K_2 K_3 K_4 / K_{D1}
\]

\[
K_{22} = K_4 + K_5 - K_4^2 (K_3 + K_4) / K_{D1} - K_5^2 (K_6 + K_7) / K_{D2}
\]

\[
K_{23} = K_{32} = -K_4 K_5 K_7 / K_{D2} \quad K_{33} = K_7 - K_7^2 (K_5 + K_6) / K_{D2}
\]

in which

\[
K_{D1} = K_2 K_3 + K_3 K_4 + K_4 K_5 + K_5 K_6 + K_6 K_7 + K_7 K_7
\]

Five independent equations can be obtained from Eq. (7). Without “shifting” or “ignoring” the 3 sensors, however, it is not possible to identify the entire stiffness matrix with 7
unknowns from these 5 independent equations. Nevertheless, additional equations can be obtained by ignoring one of the fixed sensors. There are \( \binom{3}{2} = 3 \) ways to do this, but only one of them is explained here. If the second sensor at the 4th DOF is ignored, it is possible to identify a 2 x 2 condensed stiffness matrix as follows.

\[
\mathbf{K}_2 = \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\]  

(8)

where

\[
K_{11} = K_1 + K_2 - K_2^2 (K_3 K_4 K_5 K_6 + K_4 K_4 K_5 K_7 + K_3 K_4 K_6 K_7 + K_3 K_4 K_6 K_7 + K_4 K_5 K_6 K_7) / K_{D3}
\]

\[
K_{12} = -K_2 K_3 K_4 K_5 K_6 K_7 / K_{D3}
\]

\[
K_{22} = K_7 - K_7^2 (K_3 K_4 K_5 K_6 + K_2 K_3 K_4 K_5 + K_2 K_3 K_4 K_6 + K_2 K_3 K_4 K_7 + K_2 K_3 K_5 K_6 + K_2 K_3 K_5 K_7 + K_2 K_4 K_5 K_6 + K_2 K_4 K_5 K_7)
\]

in which

\[
K_{D3} = K_2 K_3 K_4 K_5 K_6 + K_2 K_3 K_4 K_5 K_7 + K_3 K_4 K_5 K_6 K_7 + K_2 K_3 K_4 K_6 K_7 + K_2 K_3 K_4 K_7 K_7 + K_2 K_4 K_5 K_6 K_7 + K_2 K_4 K_5 K_7 K_7 + K_2 K_4 K_5 K_7 K_7
\]

In this way, 3 additional independent equations can be obtained by just ignoring the second fixed sensor. Similarly, if necessary, 3 additional independent equations can be obtained by ignoring the first or third fixed sensor. Hence, more information can be gained by ignoring one sensor (or more) at a time.

![Flowchart](image_url)

Figure 2. Flowchart for identification of story stiffness values using CMIR
The flowchart for the CMIR method is shown in Fig. 2. Suppose the total DOF of the structural model are divided into primary DOFs \( (p) \), which will be retained in the condensed models, and the secondary DOF \( (s) \), which will be deleted from the model. The mass and stiffness matrices \( M \) and \( K \) in Eq. (1) can be partitioned as

\[
\begin{bmatrix}
M_{pp} & M_{ps} \\
M_{sp} & M_{ss}
\end{bmatrix}
= \begin{bmatrix}
K_{pp} & K_{ps} \\
K_{sp} & K_{ss}
\end{bmatrix}
\tag{9}
\]

Using the condensed model, the \( n \times n \) mass and stiffness matrices can be obtained by the OKID/ERA as well as conversion from the state space model to the second-order model with \( n \) sensors. It is noted that \( n \) is the number of primary DOFs. An eigen-analysis on these condensed matrices (\( K_{pp} \) and \( M_{pp} \)) yields the identified natural frequencies and mode shapes denoted as

\[
\Lambda = \text{diag}(\Omega_1^2, \ldots, \Omega_n^2)
\quad \Phi_p = [\varphi_{p1}, \ldots, \varphi_{pn}]
\tag{10}
\]

The eigen-matrix in Eq. (10) is then expanded to include secondary DOFs using

\[
\varphi_s = -(K_{ss} - \Omega_i^2 M_{ss})^{-1}(K_{sp} - \Omega_i^2 M_{sp})\varphi_{pi}, \quad i = 1, \ldots, n
\tag{11}
\]

The System Equivalent Reduction Expansion Process (SEREP) preserves the identified eigenvalues and eigenvectors during the expansion process. The SEREP transformation matrix \( P \) is then computed from the expanded mass-normalized eigen-matrix. The stiffness and mass matrices of the condensed model refined by the SEREP method are

\[
K_{SE} = P^T K P \quad M_{SE} = P^T M P
\tag{12}
\]

The identified values are obtained from the measurement by the OKID/ERA and converted from the first-order model to the second-order model [Eq. (10)], whereas the analytical values are estimated from the SEREP [Eq. (12)]. The analytical values are updated to fit the identified values based on a trial set of stiffness parameters generated by a genetic algorithm (GA). In other words, the procedure to obtain \( K \) is to assume an initial set of values for \( K_{sp} \) and \( K_{ss} \) to be used with Eq. (11) and compute Eq. (12). The latter is then used to compute the eigenvalues and eigenvectors and determine whether they match Eq. (10). Hence, the problem of recovering the stiffness values is transformed into minimization of the difference between the identified and the analytical values [error function, Eq. (14)] and can be solved by any suitable optimization method or, in this study, by GA. GA is found to be suitable in this study because it can be used to solve highly nonlinear optimization problem. It is noted that the identified condensed stiffness and mass matrices (\( K_{pp} \) and \( M_{pp} \)) obtained from the conversion from the state space model to the second-order model are needed to form \( K \) and \( M \). Although it is possible to calculate eigenvalues and eigenvectors directly from the state space model, the optimization of eigenvalues and eigenvectors obtained from the same identified condensed stiffness and mass matrices as suggested in this study is computationally more stable.

The eigenvalues and eigenvectors from the identified and analytical condensed matrices are used in the error function. To estimate the accuracy of eigenvectors, the following Modal Assurance Criterion (MAC) [22] is used
\[ MAC(\Phi^c_i, \Phi^I_i) = \frac{\left| \Phi^c_i \cdot \Phi^I_i \right|^2}{\left| \Phi^c_i \right| \left| \Phi^I_i \right|} \]  

(13)

where superscripts ‘c’ and ‘I’ are the analytical and identified eigenvectors, respectively. Therefore, the error function is defined as

\[ \varepsilon = \sum_{i=1}^{6} \left[ (1 - \frac{\lambda_i^c}{\lambda_i^I})^2 + (1 - MAC(\Phi^c_i, \Phi^I_i))^2 \right] \]  

(14)

3. EXPERIMENTAL VERIFICATION

A small-scale laboratory model of an eight-storey steel frame is fabricated and tested to validate the applicability of the proposed method. The steel frame has a total height of 1.2 m and a plan of approximately 400 mm × 500 mm. At each storey, there are six columns, three longitudinal beams in the x-direction and four transverse beams in the y-direction. All elements have a cross section of 25 × 5 mm. In addition, steel bars of 5 mm diameter diagonally brace the beams to simulate floor rigidity. All connections are done by welding. The steel used for all members is grade 43 with density of 7850 kg/m³ and Young’s modulus of 205 GPa approximately. Figure 3 shows the experimental set-up of dynamic testing for the steel frame. The model is welded at the base to a 20-mm thick steel plate, which is bolted to a rigid supporting frame using M25 bolts.

![Experimental set up for stiffness and damage identification of steel frame model](image)

The aim is to demonstrate whether the proposed identification strategy could retrieve the full stiffness matrix from experimental condensed models and detect storey stiffness changes in terms of location and severity. In the experimental study, damage of the steel frame is
created in progression by cutting the selected columns. Each cut of column represents a reduction of approximately 16.7% of storey stiffness value at that storey. In this way, the damage created can be fairly accurately quantified by computing the decrease in stiffness value. Two damage scenarios are created in the following sequence:

Scenario 1: Two centre columns on the top and bottom sides of the steel frame at the 3\textsuperscript{rd} storey and one centre column on the top side at the 7\textsuperscript{th} storey. Thus, stiffness integrity indices for the 3\textsuperscript{rd} and 7\textsuperscript{th} storeys are 0.67 and 0.83, respectively.

Scenario 2: Scenario 1 plus the centre column on the bottom side at the 7\textsuperscript{th} storey and two centre columns on the top and bottom sides at the 4\textsuperscript{th} storey. Thus, stiffness integrity indices for the 3\textsuperscript{rd}, 4\textsuperscript{th} and 7\textsuperscript{th} storeys are 0.67 each.

### 3.1 Stiffness identification of undamaged frame

Incomplete measurement is carried out with 4 sensors and subsequently 6 sensors. Both the fixed and repositioned sensor approaches are used to extract sufficient information for incomplete measurement with 4 sensors whereas the fixed sensor approach is used with 6 sensors. The 4 sensors are located at storeys 1, 3, 5 and 8, and the 6 sensors are located at storeys 1, 3, 4, 5, 7 and 8. In the fixed sensor approach with 4 sensors, sensor at 3\textsuperscript{rd} storey is ignored, whereas the sensor at 4\textsuperscript{th} storey is ignored in the case with 6 sensors. In the case of repositioned sensor approach with 4 sensors, only one reposition is used where the sensor at the 5\textsuperscript{th} storey is repositioned to the 6\textsuperscript{th} storey.

The stiffness identification results with 4 and 6 sensors using shaker excitation is summarized in Table 1. Static experiments are conducted to determine the actual stiffness and these values are regarded as the benchmark values to be compared with. The results show that the storey stiffness values can be estimated with reasonable accuracy for both the fixed and repositioned sensor approaches. The mean error is less than 6% error for 6 sensors and about 10% for 4 sensors. The noise-to-signal ratios for the output data are in the range of 6-8%, depending on the type of accelerometers used. The noise level is determined by acquiring the time signals when no excitation is applied. The noise signal is approximately zero-mean and thus not a bias error.

<table>
<thead>
<tr>
<th>Storey</th>
<th>Identified Stiffness in N/m (Error in Bracket)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4 sensors (fixed sensor approach)</td>
</tr>
<tr>
<td>1</td>
<td>1349022 (-12.8%)</td>
</tr>
<tr>
<td>2</td>
<td>440912 (-8.5%)</td>
</tr>
<tr>
<td>3</td>
<td>369999 (-6.1%)</td>
</tr>
<tr>
<td>4</td>
<td>410294 (10.0%)</td>
</tr>
<tr>
<td>5</td>
<td>340129 (-9.9%)</td>
</tr>
<tr>
<td>6</td>
<td>348823 (-12.2%)</td>
</tr>
<tr>
<td>7</td>
<td>423456 (12.0%)</td>
</tr>
<tr>
<td>8</td>
<td>398012 (10.1%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean Absolute Error</th>
<th>Max. Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 sensors</td>
<td>10.21%</td>
<td>12.8%</td>
</tr>
<tr>
<td>4 sensors</td>
<td>7.71%</td>
<td>10.5%</td>
</tr>
<tr>
<td>6 sensors</td>
<td>5.85%</td>
<td>11.4%</td>
</tr>
</tbody>
</table>

Table 1. Identified story stiffness values with incomplete measurement
3.2 Structural damage detection

Cutting one column at a storey will affect the storey stiffness of adjacent storeys due to Vierendeel effect [23]. To account for this effect, the actual stiffness integrity indices are determined by simple static experiment, the results of which serve as the benchmark for verifying the proposed method. The identified stiffness integrity indices for Damage Scenario 1 with complete and incomplete measurement are shown in Figure 4. For Damage Scenario 1 with cut at two centre columns at the 3\(^{rd}\) storey and one centre column at the 7\(^{th}\) storey, the identified stiffness integrity index has the value of 0.68 at the 3\(^{rd}\) storey and 0.85 at the 7\(^{th}\) storey based on static experiments. The identified stiffness integrity index is 0.60 at the 3\(^{rd}\) damaged storey and 0.85 in the 7\(^{th}\) damaged storey based on the proposed method using 6 sensors. This observation shows that the damage at the 3\(^{rd}\) storey is more severe than that at the 7\(^{th}\) storey as shown in Figure 4. The mean error is 11.55% for the case of 4 sensors and 9.85% for the case of 6 sensors. However, using complete measurement, the mean error is reduced to 7.05%. Obviously with increasing number of measurement, the leakage problem is reduced and more accurate identification results are obtained.

The identified stiffness integrity indices for Damage Scenario 2 with complete and incomplete measurement are shown in Figure 5. For the Damage Scenario 2 with cut at two centre columns at the 3\(^{rd}\), 4\(^{th}\) and 7\(^{th}\) storey, all the identified stiffness integrity indices for damaged storey have almost the same value of 0.68. The identified stiffness integrity indices at the 3\(^{rd}\), 4\(^{th}\) and 7\(^{th}\) damaged storey are 0.74, 0.74 and 0.72, respectively based on the proposed method using 4 sensors. This gives a satisfactory performance with the mean and maximum error less than 8% and 11%, respectively. Figure 5 shows that the damage extent of the storey can be estimated accurately for the Damage Scenario 2.

![Figure 4. Identified story stiffness integrity indices for Damage Scenario 1](image)
4. CONCLUSIONS

This study focuses on development of a practical numerical identification strategy for off-line applications to linear structures. A methodology for identification of stiffness and damage using incomplete measurements is presented. The model condensation and mode shape expansion (SEREP) are adopted and incorporated in the OKID/ERA procedure as well as providing for the conversion from first-order to second-order model. The main research significance is that with the CMIR method, it is possible to utilize several condensed stiffness matrices to determine stiffness and stiffness integrity index of each story. It has been shown that the method retrieves the full set of stiffness parameters from identified condensed stiffness matrices by extracting sufficient information with either fixed sensor approach or repositioned sensor approach. Experimental results show that repositioned sensor approach is more accurate and effective for identifying stiffness values and structural damages than that using fixed sensor approach. By and large, the identified stiffness integrity index, in particular, is found to reveal the location and extent of damage well accounting for effects of incomplete and noisy measurement.

REFERENCES


